Exam - PoMS, 20/02/2014

- Write each question on a sheet of paper.
- Write your **name** and **student ID** on each sheet.
- Pay attention to units. A numerical result without a unit will be considered wrong!
- Only a regular calculator is allowed.
- This is NOT an open book exam.
- You are allowed to bring one A4 page with your own notes (one side only).
- You have **3 hours** to complete the exam.
- Note: $\mathscr{L}(t^n\, e^{-\alpha t}) = \frac{n!}{(s+\alpha)^{n+1}}$.

Question 1: General (2 points)

a) What is the Nyquist sampling theorem and explain its underlying principle.

b) What is reluctance and explain qualitatively the working principle of a variable reluctance tachogenerator.

c) What is the working principle behind Amplitude Modulation (AM) and explain how such a technique can help to reduce external interferences?

d) What is the working principle behind Frequency Modulation (FM) and explain how such a technique can help to reduce external interferences?

Question 2: A force measurement system (2 points)

A force measurement system consists of linear elements and has an overall steady-state sensitivity of unity. The dynamics of the system are determined by the second-order transfer function of the sensing element, which has a natural frequency $\omega_n = 35$ rad/s and a damping ratio $\xi = 0.15$. Calculate the system dynamic error corresponding to the periodic input force signal:

$$F(t) = 50\,(\sin 10t + 1/3 \sin 30t + 1/5 \sin 50t)\,,$$

with t in seconds and $F(t)$ in Newtons.

Question 3: A strain gauge measurement system (2 points)

Consider a strain gauge measurement system as indicated in Fig. 1. The sensor consists of 4 strain gauges for which R_1 and R_4 are placed in tensile mode, e.g. $R_1 = R_4 = R_0(1+Ge)$, and R_2 and R_3 in compressive mode, e.g. $R_2 = R_3 = R_0(1-Ge)$, with $R_0 = 100\ \Omega$ at a temperature of $T = 20\ ^\circ\mathrm{C}$ and a gauge factor of $G = 2$. The power supply has a voltage of $V_S = 12$ V. The sensor is connected to a recorder element via a cable with a total resistance of $R_C = 50\ \Omega$. The recorder has a loading resistance of $R_L = 10\ \mathrm{k}\Omega$.

a) Find the Thévenin equivalent voltage, V_{Th}, and the corresponding impedance, Z_{Th}, of the sensor element.

b) Calculate the voltage over R_L, V_L, for a strain of $e = 10^{-3}$ at $T = 20\ ^\circ\mathrm{C}$. How large is the loading effect on the recorder?

c) The temperature of the sensor increases, which leads to an increase in the resistances of the strain elements, R_1, R_2, R_3, and R_4, by 5 Ω. What is the main underlying physical mechanism that increases the resistance of a conductor due to an increase in temperature? Discuss the influence of this effect on V_L.

d) The output voltage on the recorder, V_L, is considered to be too small. Redesign the measurement system in such a way that V_L is amplified by a factor 10. For this, introduce an ideal operational amplification element without changing the sensor and recorder elements and their parameters.

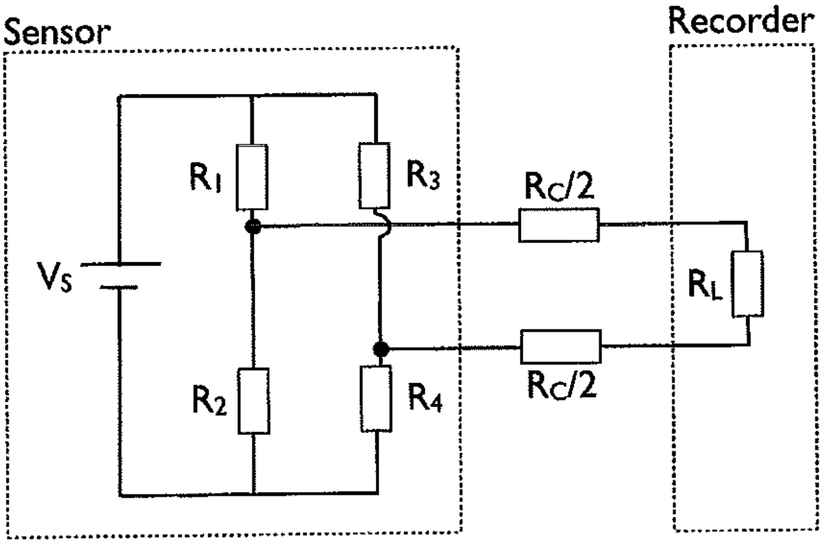


Figure 1: Figure corresponding to question 3.

Question 5: A current transmitter (2 points + 1 bonus point)

Consider a current transmitting system as indicated in Fig. 3. A sensor consists of a Norton current source connected to a resistance R_N=1 MΩ and a capacitor C_N=1000 pF. The indicator reads the voltage (V_L) over a resistance R_L=10 kΩ. An interfering series mode voltage V_{SM} can be present on the system.

a) Take V_{SM}=0. Show that the transfer function is given by

$$\frac{\Delta \tilde{V}_L}{\Delta \tilde{i}_N}(s) = R' \left(\frac{1}{1+s\tau} \right), \tag{1}$$

with $R'=R_N R_L/(R_N+R_L)$=9.9 kΩ and $\tau=C_N R'$=9.9 μs.

b) The current i_N increases suddenly (as a step function) to 1 mA at t=0 from a steady-state condition with $i_N=V_L$=0 at a time $t<0$. Give an expression for $V_L(t)$ and make a sketch of its time dependence.

c) Consider a steady-state situation with an interfering voltage V_{SM}=10 V together with i_N=1 mA. How large is the influence of the interfering voltage on V_L? Give the signal-to-noise ratio (S/N) in dB.

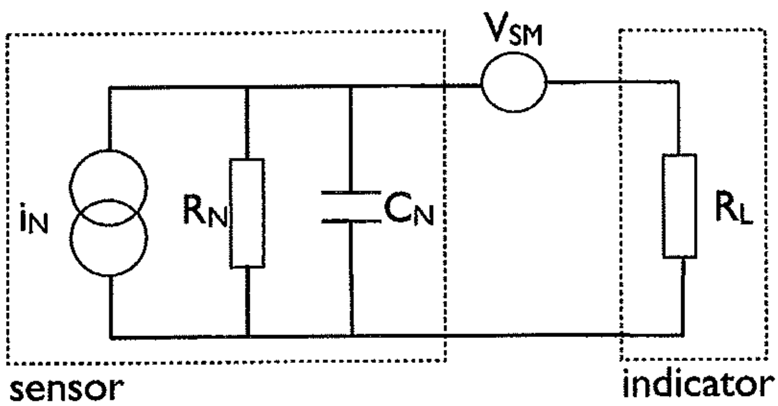


Figure 3: A current transmitter.

Question 4: A potentiometric displacement sensor (2 points)

A potentiometer has a total length of d_T and a resistance of R_P. The supply voltage of the potentiometer is indicated as V_S. The potentiometer is connected to a recorder with a resistance R_L. The setup is illustrated in Fig. 2.

a) Show that the voltage at the recorder, V_L, is given by

$$V_L = V_S \frac{x}{(R_P/R_L)x(1-x)+1},$$

with $x = d/d_T$ and d is the displacement. [Hint: you can make use of the Thévenin equivalent circuit law.]

b) In the ideal case, $R_L >> R_P$, we would have the relation $V_L/V_S \approx x$. Estimate the maximum non-linearity of V_L/V_S for R_L=1 kΩ and R_P=100 Ω. At which value of x does one find the maximum non-linearity?

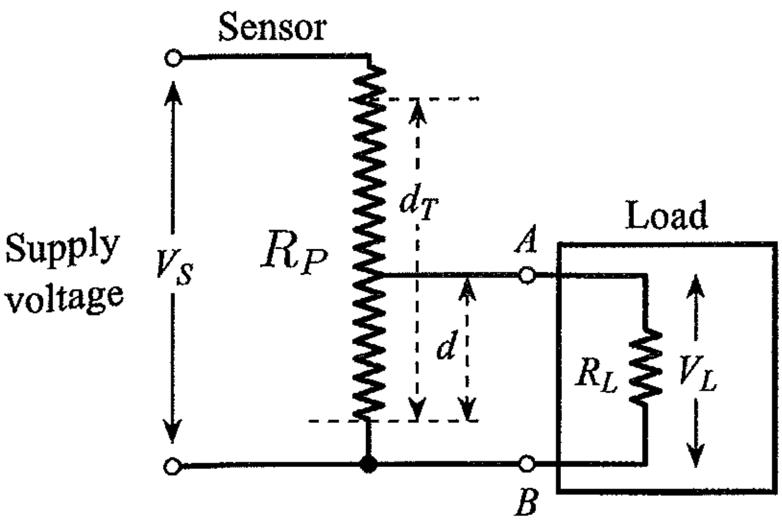


Figure 2: A potentiometric displacement sensor.

Steady-state sensitivity of 1 : $\frac{1}{k} = 1 \left(\frac{m}{N}\right)$

natural frequency : $\omega_n = 35 \left(\frac{rad}{s}\right)$

damping ratio : $\xi = 0.15$

2

$$E(t) = F_{out}(t) - F_{in}(t)$$

From page 63 of Bentley : $F_{out}^{(n)}(t) = \hat{I}^{(n)} \cdot \overset{1}{(k)} \cdot |G(j\omega_n)| \cdot \sin(\omega_n t + \phi_n)$

$$\Rightarrow F_{out}(t) = 50\left[|G(10j)| \sin(10t + \phi_{10}) + \frac{|G(30j)|}{3} \sin(30t + \phi_{30}) + \frac{|G(50j)|}{5} \sin(50t + \phi_{50}) \right]$$

Second-order transfer function :

$$\begin{cases} G(s) = \dfrac{1}{\frac{1}{\omega_n^2} s^2 + \frac{2\xi}{\omega_n} s + 1} \\[2ex] |G(j\omega)| = \dfrac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2 \left(\frac{\omega}{\omega_n}\right)^2}} \end{cases}$$

$$\Rightarrow |G(10j)| = \frac{1}{\sqrt{\left(1 - \frac{10^2}{35^2}\right)^2 + 4 \times 0.15^2 \times \left(\frac{10}{35}\right)^2}} = 1.08$$

$$\& \ \phi_{10} = \arg G(j\omega)\Big|_{\omega = 10\left(\frac{rad}{s}\right)} = -\arctan\left[\frac{2\xi \frac{\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2}}\right]_{\omega = 10\left(\frac{rad}{s}\right)} = -5.33^\circ$$

$-0,093$

Finally, we will have : $|G(30j)| = 2.71$, $|G(50j)| = 0.89$

, $\phi_{30} = -44.1^\circ$, $\phi_{50} = -157.6^\circ$

$-0,77$ $+0,39$

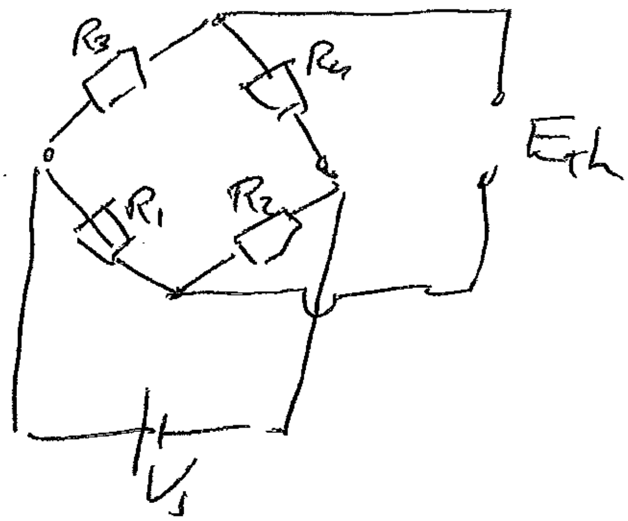
Thus,

$$E(t) = 50\left[1.08 \sin(10t - 5^\circ) - \sin(10t)\right] + \frac{50}{3}\left[2.71 \sin(30t - 44^\circ) - \sin(30t)\right]$$
$$+ \frac{50}{5}\left[0.89 \sin(50t - 158^\circ) - \sin(50t)\right]$$

$1\frac{1}{2}$

5 a)

sensor =
(wheatstone bridge)



3

$$E_{Th} = V_s\left[\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right] = V_s\left[\frac{R_0^2(1+Ge)^2 - R_0^2(1-Ge)^2}{\cancel{4}R_0^2}\right]$$

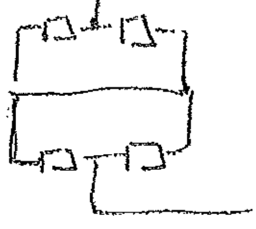
$$= V_s\left[\frac{4Ge}{\cancel{4}}\right] = \underline{V_s Ge}$$

$$Z_{Th} = \frac{R_3R_4}{R_3+R_4} + \frac{R_1R_2}{R_1+R_2} = \frac{R_0^2(1-Ge)(1+Ge)}{2R_0} + \frac{R_0^2(1+Ge)(1-Ge)}{2R_0}$$

$$= R_0(1-Ge)(1+Ge) = \underline{\underline{R_0\left[1-(Ge)^2\right]}}$$

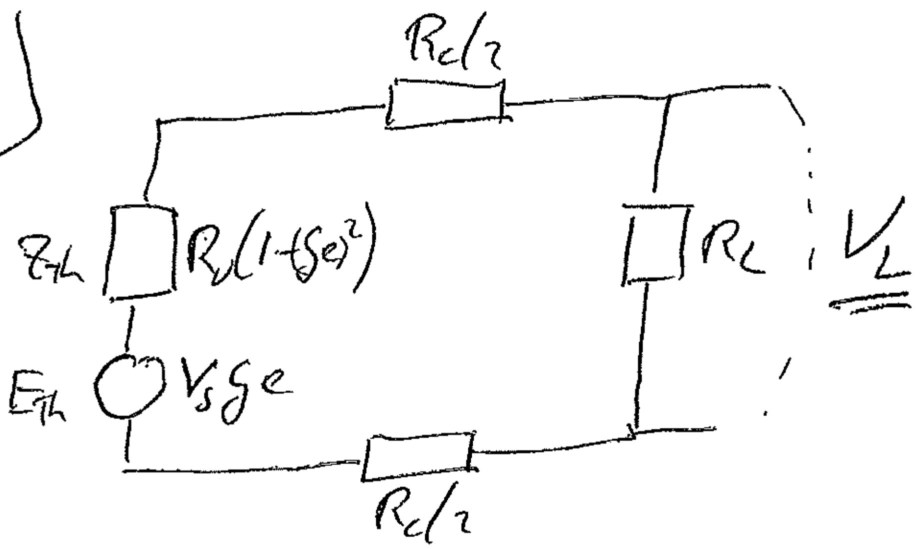
1/2

$$\frac{R_2}{R_2+R_1} - \frac{R_4}{R_3+R_4}$$
$$= \frac{R_2(R_3+R_4) - R_4(R_2+R_1)}{(R_1+R_2)(R_3+R_4)}$$
$$= R_3R_2 - R_1R_4$$



$$\checkmark\ E_{Th} = \frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}$$

5 b)



$$V_L = E_{Th} \frac{R_L}{R_L + R_c + Z_{Th}} = V_s g e \frac{10 \cdot 10^3 \ \Omega}{10 \cdot 10^3 + 50 + 100 \ \Omega}$$

$$= V_s \cdot g \cdot e \cdot 0{,}985 = \underline{\underline{23.6 \text{ mV}}}$$

loading effect: $1 - 0{,}985 = \underline{\underline{0.015}} = \underline{\underline{1.5\%}}$

$\frac{1}{2}$

5 c) temp. increase of conductor => τ reduces => $R \uparrow$
(time between interactions smaller)

$$\boxed{R = \rho \frac{l}{A} = \frac{m_e}{e^2 n \tau} \frac{l}{A}}$$

temperature effect only => $R_1 = R_2 = R_3 = R_4 = R_0 (1 + \alpha \Delta T)$

$$E_{Th} \text{ from } \Delta T \Rightarrow E_{Th} = \frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} = \underline{\underline{0}}$$

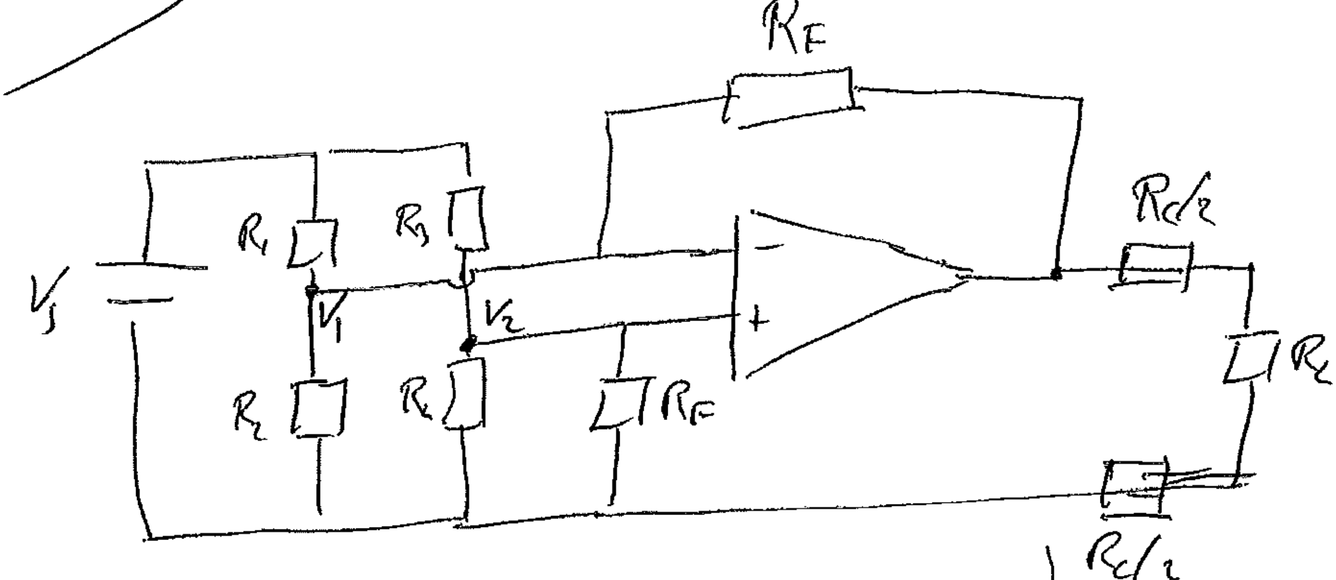
$$Z_{Th} \text{ from } \Delta T \Rightarrow Z_{Th} = \frac{R_3 R_4}{R_3 + R_4} + \frac{R_1 R_2}{R_1 + R_2}$$

$$= R_0 (1 + \alpha \Delta T) = 105\ \Omega$$

~~an~~ => hardly changes $\underline{\underline{V_L}}$!

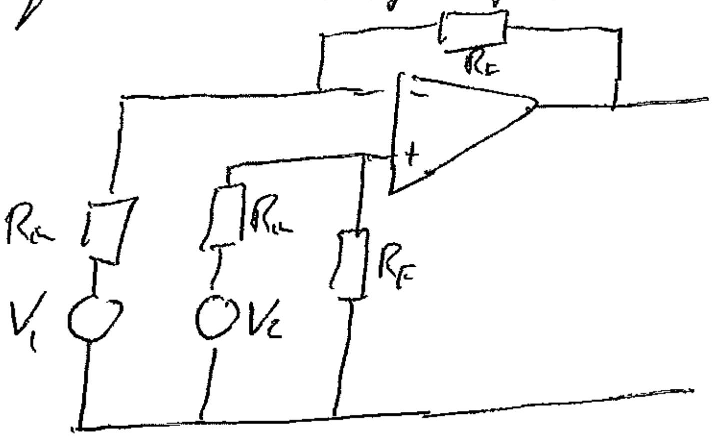
1

5 d



equivalent to diff. amplifier

1



$$V_{out} = (V_2 - V_1)\frac{R_F}{R_{in}}$$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_0}{2} \qquad [(Se)^2 \text{ small!}]$$

$$V_1 = V_s \frac{R_2}{R_1 + R_2} = V_s \frac{R_0(1 - Se)}{2R_0} = \frac{V_s}{2}(1 - Se)$$

$$V_2 = V_s \frac{R_4}{R_3 + R_4} = \cdots \frac{(1+Se)}{} = \frac{V_s}{2}(1 + Se)$$

cont'd

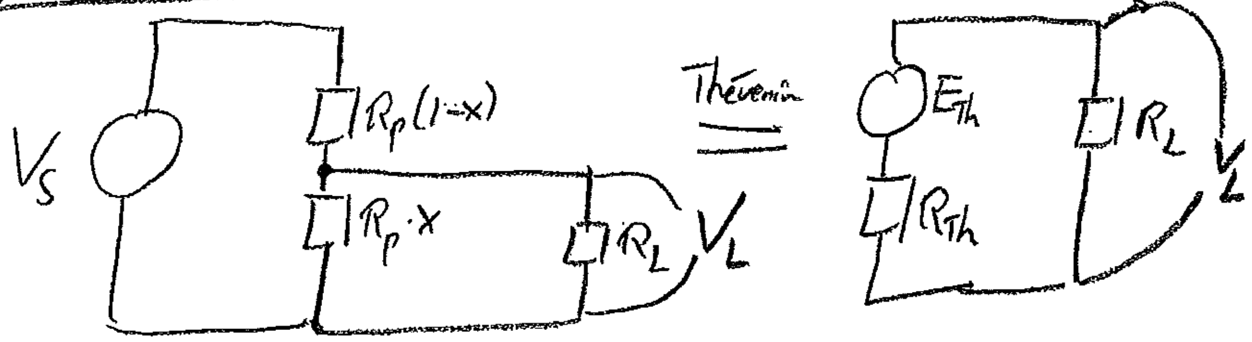
$$\Rightarrow V_{out} = \frac{2R_F}{R_U} V_s G e$$

factor 10 Amplification, $\Rightarrow \dfrac{2R_F}{R_U} = 10$

$\hookrightarrow R_F = 5R_U = 500\,\Omega$

Question 4

equivalent circuit:

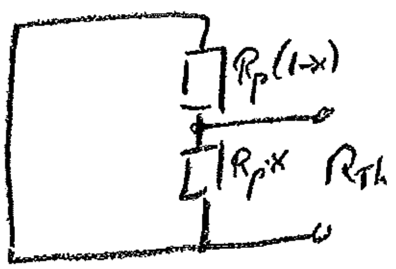


a)

E_{Th} : open circuit ($R_L \to \infty$)

$$E_{Th} = V_s \frac{R_p \cdot x}{R_p \cdot x + R_p(1-x)} = \underline{\underline{V_s \cdot x}}$$

R_{Th} : short cut V_s :



$$\Rightarrow R_{Th} = R_p(1-x) \,//\, R_p \cdot x$$

$$\Rightarrow \frac{1}{R_{Th}} = \frac{1}{R_p(1-x)} + \frac{1}{R_p \cdot x}$$

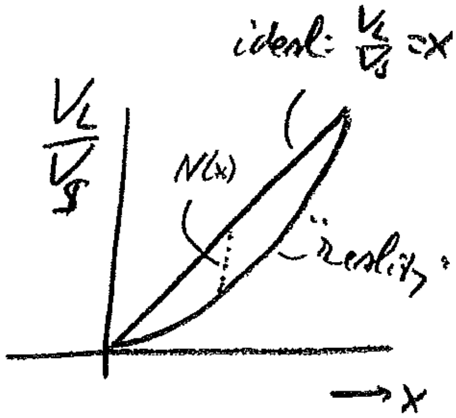
$$\Rightarrow R_{Th}^{-1} = \frac{R_p x + R_p(1-x)}{R_p^2(1-x)x} \Rightarrow \underline{\underline{R_{Th} = R_p x(1-x)}}$$

Hence,

$$V_L = E_{Th} \frac{R_L}{R_L + R_{Th}} = V_s \cdot x \frac{R_L}{R_L + R_p x(1-x)} = \underline{\underline{V_s \frac{x}{\left(\frac{R_p}{R_L}\right) x(1-x) + 1}}}$$

Question 4, cont$^{\underline{d}}$

b) $$\frac{V_L}{V_s} = \frac{x}{\underbrace{\left(\frac{R_p}{R_L}\right)}_{=0.1} x(1-x) + 1} \Rightarrow$$



<u>non-linearity of V_L/V_s</u> :

$$N(x) = x - \frac{V_L}{V_s} = x\left[1 - \frac{1}{0.1x(1-x)+1}\right]$$

$$= x\left[\frac{0.1x(1-x)+1}{0.1x(1-x)+1} - \frac{1}{0.1x(1-x)+1}\right] = x\frac{0.1x(1-x)}{\underbrace{0.1x(1-x)}_{\ll 1}+1}$$

$$= 0.1x^2(1-x) = 0.1\left[x^2 - x^3\right]$$

<u>maximum non-linearity</u>

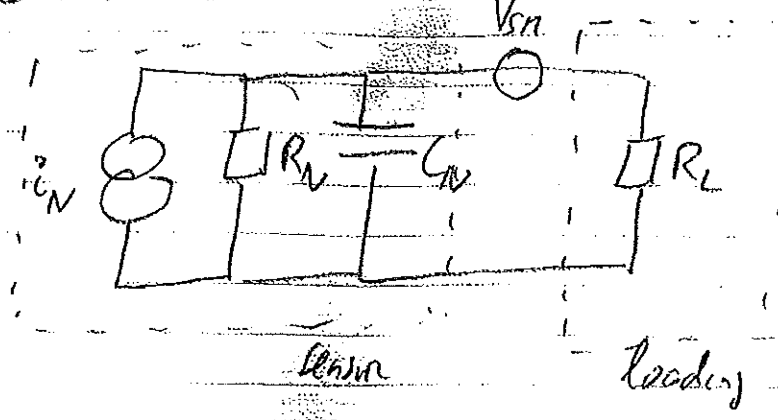
$$\frac{dN}{dx} = 0.1\left[2x - 3x^2\right] = 0 \Rightarrow x = \frac{2}{3}$$

$$N_{max} = N\left(x = \frac{2}{3}\right) = 0.1\left[\frac{4}{9} - \frac{8}{27}\right] = 0.1\frac{4}{27} = \frac{4}{270}$$

$$= \underline{\underline{1.48\,\%}}$$

Q5

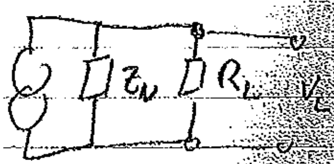
<u>CURRENT TRANSMITTER</u>



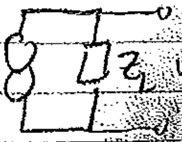
$R_L = 10\,k\Omega$
$R_N = 1\,M\Omega$
$C_N = 1000\,pF$

(a) $\frac{\Delta \tilde{V}_L}{\Delta \tilde{i}_N}(s)$ with $V_{sn} = 0$

$$Z_N = R_N // C_N = \frac{R_N \frac{1}{sC_N}}{R_N + \frac{1}{sC_N}} = \frac{R_N}{sC_N R_N + 1}$$



$$Z_L = R_L // Z_N = \frac{Z_N R_L}{Z_N + R_L} = \frac{\frac{R_N R_L}{sC_N R_N + 1}}{\frac{R_N}{sC_N R_N + 1} + R_L} = \frac{R_N R_L}{R_N + R_L + sC_N R_N R_L}$$



$$= \frac{R_N R_L}{R_N + R_L}\left[\frac{1}{1 + sC_N\left(\frac{R_N R_L}{R_N + R_L}\right)}\right] = \underbrace{R'\left(\frac{1}{1+s\tau}\right)}_{= G(s)}$$

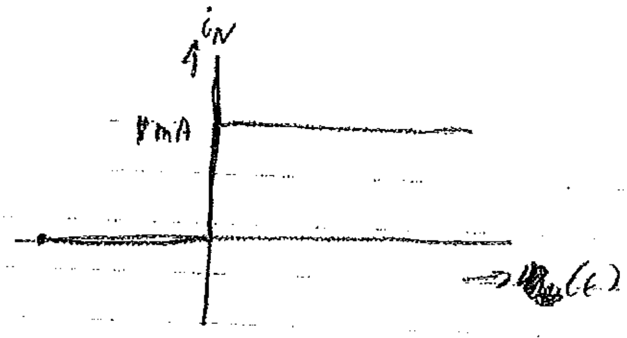
with $\tau = C_N R'$

$$R' = \frac{R_N R_L}{R_N + R_L}$$

$R' \approx R_L = 10\,k\Omega$ $(= 9.9\,k\Omega)$

$\tau = C_N R' \approx 10^{-5}\,s = \underline{10\,\mu s}$ $(= 9.9\,\mu s)$

(b)



$$i_N(t) = \mu(t) \cdot 1 \text{ mA}$$

$$\Delta \tilde{i}_N(s) = \frac{1}{s} \cdot 1 \text{ mA}$$

$$\Delta \tilde{V}_L = \Delta \tilde{i}_N \cdot \frac{R'}{1+\tau s}$$

$$= \underbrace{\frac{1}{s} \cdot \frac{1}{1+\tau s}} \cdot 9.9V$$

$$= \frac{A}{s} + \frac{B}{1+\tau s} \qquad \Rightarrow A(1+\tau s) + Bs = 1$$

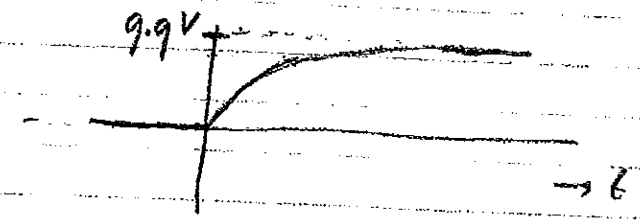
$$= \frac{1}{s} - \frac{\tau}{1+\tau s} \qquad A + (A\tau + B)s = 0$$

(for all s)

$$= \frac{1}{s} - \frac{1}{\frac{1}{\tau} + s} \qquad \Rightarrow A = 1 \text{ and } A\tau + B = 0$$

$$\Rightarrow B = -A\tau = -\tau$$

$$\xrightarrow{\mathcal{L}^{-1}} \Delta V_L(t) = 9.9\,\mu(t)\left[1 - e^{-t/\tau}\right] \quad [V]$$



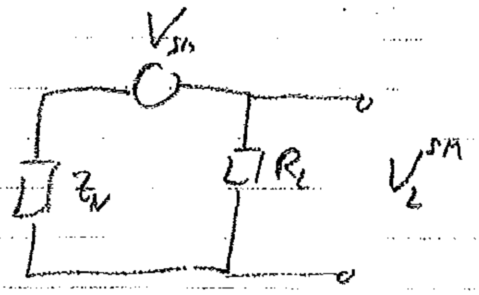
~~(c) bandwidth $0 \leftrightarrow \frac{1}{\tau} = 10^5 \frac{rad}{s} = \frac{10^5}{2\pi}$ Hz~~

~~$\Downarrow$ ω_b $\quad \Downarrow$ f_b ($= \omega_b / 2\pi$)~~

c)

(a) $V_{sm} = 10\,V$

disturbance on V_L : V_L^{sm}
(superposition)



$$V_L^{sm} = V_{sm} \cdot \frac{R_L}{R_L + Z_N} \overset{\text{steady state}}{=} V_{sm} \frac{R_L}{R_L + R_N} = \frac{V_{sm}}{101} = 0.1\,V$$

$$V_L = 9.9 + 0.1\,V = 10\,V$$

superposition

$$\frac{S}{N} = 20 \log_{10} \left(\frac{10\,V}{0.1\,V} \right) = 40\,dB.$$